GRE MATH FORMULA SHEET

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NUMBERS

FRACTIONS TO DECIMALS

$\frac{1}{2} = 0.5$	$\frac{1}{5} = 0.2$	$\frac{1}{6} = 0.1\bar{6}$
$\frac{1}{3} = 0.\bar{3}$	$\frac{2}{5} = 0.4$	$\frac{5}{6} = 0.8\bar{3}$
$\frac{2}{3} = 0.\bar{6}$	$\frac{3}{5} = 0.6$	$\frac{1}{8} = 0.125$
$\frac{1}{4} = 0.25$	$\frac{4}{5} = 0.8$	$\frac{3}{8} = 0.375$
$\frac{3}{4} = 0.75$	$\frac{1}{20} = 0.05$	$\frac{5}{8} = 0.625$
$\frac{1}{10} = 0.1$	$\frac{1}{25} = 0.04$	$\frac{7}{8} = 0.875$

Know the decimal to fraction conversion for the same set of numbers, i.e. $0.375 = \frac{3}{8}$.

POWERS OF NUMBERS

OTHER NUMBERS

$\pi \approx 3.14$	$0! = 1$	$4! = 24$
$\sqrt{2} \approx 1.4$	$1! = 1$	$5! = 120$
$\sqrt{3} \approx 1.7$	$2! = 2$	$6! = 720$
$\sqrt{5} \approx 2.2$	$3! = 6$	

Notice the following:

$1.4^2 = 1.96 \approx 2 \qquad 1.7^2 = 2.89 \approx 3$

$1\,million = 1,000,000 = 10^6$

$1\,billion = 1,000,000,000 = 10^9$

Prime numbers less than 20:

2, 3, 5, 7, 11, 13, 17, 19

DIVISIBILITY

PRIME NUMBER

EVEN & ODD

Even ± Even = Even Even ± Odd = Odd

Odd ± Odd = Even

Even · Anything = Even Odd · Odd = Odd

Recognize

n is a positive integer.

Examples of even numbers: $2n$, $4+2n$, $n(n+1)$, $10(n+3)$, $n+n^3+2$, n^3+n^2

Examples of odd numbers: $2n+1$, $4n^2-1$, n^2+n+1, $(6n-9)(2n^3+3)$, $2n^2+4n-3$

ARITHMETIC

ORDER OF OPERATIONS

1. Parentheses.
2. Exponents.
3. Multiplication and Division from left to right.
4. Addition and Subtraction from left to right.

FRACTIONS

Adding Fractions

$$\frac{a}{b}+\frac{c}{d} = \frac{a\times d}{b\times d}+\frac{c\times b}{d\times b} = \frac{a\times d + c\times b}{b\times d}$$

Ex: $\frac{2}{3}+\frac{3}{4} = \frac{2\times 4}{3\times 4}+\frac{3\times 3}{4\times 3} = \frac{2\times 4+3\times 3}{4\times 3} = \frac{17}{12}$

Subtracting Fractions

$$\frac{a}{b}-\frac{c}{d} = \frac{a\times d}{b\times d}-\frac{c\times b}{d\times b} = \frac{a\times d - c\times b}{b\times d}$$

Ex: $\frac{1+y}{y}-\frac{x-1}{x} = \frac{x(1+y)}{xy}-\frac{y(x-1)}{yx} = \frac{x(1+y)-y(x-1)}{xy}$

$= \frac{x+xy-yx+y}{xy} = \frac{x+y}{xy}$

Multiplying Fractions

$$\frac{a}{b}\times\frac{c}{d} = \frac{a\times c}{b\times d}$$

Cancel any common factors from the denominators and the numerators and then multiply.

Ex: $\frac{5}{6}\times\frac{3}{10} = \frac{1\times 1}{2\times 2} = \frac{1}{4}$

Dividing Fractions

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b}\times\frac{d}{c}$$

Ex: $\frac{\frac{5}{8}}{\frac{3}{4}} = \frac{5}{8}\times\frac{4}{3} = \frac{5}{6}$

Multiplying/Dividing Fractions with Decimals

Convert decimals into fractions, and then calculate.

Ex: $0.33\times\frac{2}{11}\div 0.6 = \frac{33}{100}\times\frac{2}{11}\div\frac{3}{5} = \frac{33}{100}\times\frac{2}{11}\times\frac{5}{3} = \frac{1}{10}$

MULTIPLYING/DIVIDING LARGE AND SMALL NUMBERS

Express the numbers using scientific notation. Then calculate.

Ex: $\frac{0.0004}{0.008}\times 500 = \frac{4\times 10^{-4}}{8\times 10^{-3}}\times 5\times 10^2 = \frac{1\times 10^{-1}\times 5\times 10^2}{2} = 25$

MULTIPLYING/DIVIDING EXPONENTIALS

RADICALS

ALGEBRA

EXPONENTS

COMMON PRODUCTS

$(a+b)(c+d) = ac+ad+bc+bd$

Ex: $(2x+y)(3x-2y)$
$= 2x\cdot 3x + 2x\cdot(-2y) + y\cdot 3x + y\cdot(-2y)$
$= 6x^2 - 4xy + 3xy - 2y^2$
$= 6x^2 - xy - 2y^2$

$(x+y)^2 = x^2+2xy+y^2$

Ex: $(5x+2y)^2 = (5x)^2 + 2\cdot 5x\cdot 2y + (2y)^2$
$= 25x^2 + 20xy + 4y^2$

$(x-y)^2 = x^2-2xy+y^2$

Ex: $(\sqrt{5}-\sqrt{3})^2$
$= (\sqrt{5})^2 - 2\cdot\sqrt{5}\cdot\sqrt{3} + (\sqrt{3})^2$
$= 5 - 2\sqrt{15} + 3 = 8 - 2\sqrt{15}$

$(x+y)(x-y) = x^2-y^2$

Ex: $(3-\sqrt{5})(3+\sqrt{5}) = 3^2 - (\sqrt{5})^2 = 9 - 5 = 4$

Be able to apply the above rules in reverse order.

Ex: $x^2-2x-24 = (x+4)(x-6)$

$9y^2+4x^2+12xy = (3y+2x)^2$

$-8x+2x^2+8 = 2(-4x+x^2+4) = 2(2-x)^2$

$-9+4x^2 = 4x^2-9 = (2x+3)(2x-3)$

EQUATIONS WITH ONE VARIABLE

ALGEBRA (CONTINUED)

SIMULTANEOUS LINEAR EQUATIONS

Solving by Substitution

1. From any of the two equations, write y in terms of x.
2. Plug the expression for y into the other equation.
3. Solve for x in the new one-variable linear equation.
4. Compute y by plugging the value of x into the expression found in step 2.

Ex: $\begin{cases} x + y = 3 & (1) \\ x - y = 1 & (2) \end{cases}$

Using equation (1) to write y in terms of x gives $y = 3 - x$. Plugging into equation (2) gives $x - (3 - x) = 1$. Solving x yields $x = 2$. Plugging in the expression for y gives $y = 1$.

Solving by Adding or Subtracting Equations

The goal is to form a new one-variable equation by adding or subtracting the two original equations.

- If the coefficients of a variable are the same in the two equations, subtract the two equations.
- If the coefficients of a variable are of different signs in the two equations, add the two equations.
- Otherwise, match the coefficients on one variable in both equations by multiplying the proper factor, then add or subtract the two equations.

The new equation will only involve one variable. Solve it. Plug the solution back into any one of the original equations to solve for the other variable.

Ex: $\begin{cases} 3x - 2y = 4 & (1) \\ 2x - y = 3 & (2) \end{cases}$

Multiply equation (1) by 2 on both sides.
Multiply equation (2) by 3 on both sides.

$$\begin{cases} 6x - 4y = 8 & (3) \\ 6x - 3y = 9 & (4) \end{cases}$$

Subtracting equation (4) from equation (3) gives $-y = -1$. Solving for y gives $y = 1$. Plugging back into equation (1) obtains $x = 2$.

QUADRATIC EQUATIONS

EXPRESSION SUBSTITUTIONS

INEQUALITIES

ABSOLUTE VALUES

COORDINATES

GEOMETRY

TRIANGLES

CIRCLES

GEOMETRY (CONTINUED)

ANGLES

MEASUREMENTS

WORD PROBLEMS

RATE PROBLEMS

Two Objects Traveling in Opposite Directions

$$\text{Time} = \frac{\text{Total Distance Traveled}}{\text{Total Rate}}$$

where the total distance is the sum of the distance the two objects traveled, and total rate is the sum of the two rates.
Let D_T denote the total distance traveled, and R_T denote the total rate. We have the following three relations.

$$T = \frac{D_T}{R_T} \qquad D_T = R_T T \qquad R_T = \frac{D_T}{T}$$

Ex: Car A and Car B are 100 miles apart on along a route. Car A is traveling at a constant rate of 30 miles/hour, whereas car B is traveling at a constant rate of 20 miles/hour. If both cars are traveling toward each other, how long would it take for them to meet?

Let R_A and R_B denote the rates of car A and car B respectively, D_A and D_B denote the distance car A and car B traveled respectively. From the problem, we have the following:

$$R_A = 30 \qquad R_B = 20$$

$$D_T = D_A + D_B = 100$$

$$R_T = R_A + R_B = 30 + 20 = 50$$

$$T = \frac{D_T}{R_T} = \frac{100}{50} = 2$$

It would take them 2 hours to meet.

WORD PROBLEMS (CONTINUED)

RATIO/PERCENTAGE PROBLEMS

Relationship between Percentage and Ratio
Essentially, ratios and percentages convey the equivalent information. That is given the ratio between Q_1 and Q_2, we can find what percent Q_2 is of Q_1, what percent is Q_1 more/less than Q_2. Given Q_1 is x% more/less than Q_2, we can find the ratio of the two quantities.

Ex: If Q_1:Q_2 = 3 : 4, what percent is Q_1 less than Q_2?

$$\frac{Q_1}{Q_2} = \frac{3}{4} \Rightarrow Q_1 = \frac{3}{4}Q_2 = 0.75Q_2 = (1 - 0.25)Q_2$$

Q_1 is 25% less than Q_2.

Ex: If Q_1 is 10% more than Q_2, what is the ratio of Q_1 to Q_2?

$$Q_1 = (1 + 0.1)Q_2 = 1.1Q_2 = \frac{11}{10}Q_2 \Rightarrow \frac{Q_1}{Q_2} = \frac{11}{10}$$

The ratio of Q_1 to Q_2 is 11:10.

QUANTITATIVE COMPARISON STRATEGIES

STATISTICS, COUNTING, AND PROBABILITY

STATISTICS

COUNTING

PROBABILITY